Solitons in Armchair and Zigzag Geometries in the Nonlinear Dirac Equation

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We present solitons which solve the one-dimensional (1D) zigzag and armchair nonlinear Dirac equation (NLDE) for a Bose-Einstein condensate (BEC) in a honeycomb optical lattice [1], where the two types of NLDEs correspond to the two independent directions in analogy to the narrowest of graphene nanoribbons. We analyze the solution space of the 1D NLDE by finding fixed points, delineating the various regions in solution space, and through a conservation equation which we obtain as a first integral of the NLDE. For both the zigzag and armchair geometries we obtain soliton solutions using five different methods: by direct integration; through the conservation equation; by parametric transformation; a series expansion; and by the method of numerical shooting. We interpret our solitons as domain walls in 1D which separate distinct regions of pseudospin-1/2 with $S_z = \pm 1/2$, where the domain wall is topologically protected. By solving the relativistic linear stability equations (RLSE) we obtain the low-energy spectrum for excitations in the bulk region far from the soliton core and for bound states in the core. We find that excitations occur as quadrupolar pseudospin waves and as a Nambu-Goldstone mode. For a BEC of ^{87}Rb atoms, we find that our soliton solutions are stable on time scales relevant to experiments [2].

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[2] L. H. Haddad, C. M. Weaver, and Lincoln D. Carr. The nonlinear Dirac equation in Bose-Einstein condensates: Relativistic solitons in armchair and zigzag geometries. *Phys. Rev. A,* (2012), under review.